Math Tutor — Weighted Averages and Atomic Mass

You have learned that the mass of a proton is about 1 u and that a neutron is only slightly heavier. Because atomic nuclei consist of whole numbers of protons and neutrons, you might expect that the atomic mass of an element would be very near a whole number. However, if you look at the periodic table, you will see that the atomic masses of many elements lie somewhere between whole numbers. In fact, the atomic masses listed on the table are *average* atomic masses. The atomic masses are averages because most elements occur in nature as a specific mixture of isotopes. For example, 75.76% of chlorine atoms have a mass of 34.969 u, and 24.24% have a mass of 36.966 u. If the isotopes were in a 1:1 ratio, you could simply add the masses of the two isotopes together and divide by 2. However, to account for the differing abundance of the isotopes, you must calculate a *weighted average*. For chlorine, the weighted average is 35.45 u. The following two examples demonstrate how weighted averages are calculated.

Sample Problem

A sample of naturally occurring silver consists of 51.839% Ag-107 (atomic mass 106.905 093 u) and 48.161% Ag-109 (atomic mass 108.904 756 u). What is the average atomic mass of silver?

To find average atomic mass, convert each percentage to a decimal equivalent and multiply by the atomic mass of the isotope.

$$\begin{aligned} 0.518\,39 \times 106.905\,093\ \text{u} &= 55.419\ \text{u} \\ \underline{0.481\,61 \times 108.904\,756\ \text{u} = 52.450\ \text{u}} \\ 107.869\ \text{u} \end{aligned}$$

Adding the masses contributed by each isotope gives an average atomic mass of 107.869 u. Note that this value for the average atomic mass of silver is very near the one given in the periodic table.

A sample of naturally occurring magnesium consists of 78.99% Mg-24 (atomic mass 23.985 042 u), 10.00% Mg-25 (atomic mass 24.985 837 u), and 11.01% Mg-26 (atomic mass 25.982 593 u). What is the average atomic mass of magnesium?

Again, convert each percentage to a decimal and multiply by the atomic mass of the isotope to get the mass contributed by each isotope.

$$\begin{aligned} 0.7899 \times 23.985\,042\ \text{u} &= 18.95\ \text{u} \\ 0.1000 \times 24.985\,837\ \text{u} &= 2.499\ \text{u} \\ \underline{0.1101 \times 25.982\,593\ \text{u} = 2.861\ \text{u}} \\ 24.31\ \text{u} \end{aligned}$$

Adding the masses contributed by each isotope gives an average atomic mass of 24.31 u.

Practice

1. Rubidium occurs naturally as a mixture of two isotopes, 72.17% Rb-85 (atomic mass 84.911 792 u) and 27.83% Rb-87 (atomic mass 86.909 186 u). What is the average atomic mass of rubidium?
2. The element silicon occurs as a mixture of three isotopes: 92.22% Si-28, 4.69% Si-29, and 3.09% Si-30. The atomic masses of these three isotopes are as follows: Si-28 = 27.976 926 u, Si-29 = 28.976 495 u, and Si-30 = 29.973 770 u.

   Find the average atomic mass of silicon.

CHAPTER 4 Summary

BIG IDEA An atom's nucleus is surrounded by electrons that have both a wave and a particle-like behavior.

SECTION 1 The Development of a New Atomic Model

- In the early twentieth century, light was determined to have a dual wave-particle nature.
- Quantum theory was developed to explain observations such as the photoelectric effect and the line-emission spectrum of hydrogen.
- Quantum theory states that electrons can exist only at specific atomic energy levels.
- When an electron moves from one main energy level to a main energy level of lower energy, a photon is emitted. The photon's energy equals the energy difference between the two levels.
- An electron in an atom can move from one main energy level to a higher main energy level only by absorbing an amount of energy exactly equal to the difference between the two levels.

KEY TERMS

electromagnetic radiation
electromagnetic spectrum
wavelength
frequency
photoelectric effect
quantum
photon
ground state
excited state
line-emission spectrum
continuous spectrum

SECTION 2 The Quantum Model of the Atom

- In the early twentieth century, electrons were determined to have a dual wave-particle nature.
- The Heisenberg uncertainty principle states that it is impossible to determine simultaneously the position and velocity of an electron or any other particle.
- Quantization of electron energies is a natural outcome of the Schrödinger wave equation, which describes the properties of an atom's electrons.
- An orbital, a three-dimensional region around the nucleus, shows the region in space where an electron is most likely to be found.
- The four quantum numbers that describe the properties of electrons in atomic orbitals are the principal quantum number, the angular momentum quantum number, the magnetic quantum number, and the spin quantum number.

KEY TERMS

Heisenberg uncertainty principle
quantum theory
orbital
quantum number
principal quantum number
angular momentum quantum number
magnetic quantum number
spin quantum number

SECTION 3 Electron Configurations

- The ground-state electron configuration of an atom can be written by using the Aufbau principle, Hund's rule, and the Pauli exclusion principle.
- Electron configurations can be depicted by using different types of notation. In this book, three types of notation are used: orbital notation, electron-configuration notation, and noble-gas notation.
- Electron configurations of some atoms, such as chromium, deviate from the predictions of the Aufbau principle, but the ground-state configuration that results is the configuration with the minimum possible energy.

KEY TERMS

electron configuration
Aufbau principle
Pauli exclusion principle
Hund's rule
noble gas
noble-gas configuration

CHAPTER 4 Review

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SECTION 1

The Development of a New Atomic Model

REVIEWING MAIN IDEAS

1. **a.** List five examples of electromagnetic radiation.
   b. What is the speed of all forms of electromagnetic radiation in a vacuum?

2. Prepare a two-column table. List the properties of light that can best be explained by the wave theory in one column. List those best explained by the particle theory in the second column. You may want to consult a physics textbook for reference.

3. What are the frequency and wavelength ranges of visible light?

4. List the colors of light in the visible spectrum in order of increasing frequency.

5. In the early twentieth century, what two experiments involving light and matter could not be explained by the wave theory of light?

6. **a.** How are the wavelength and frequency of electromagnetic radiation related?
   b. How are the energy and frequency of electromagnetic radiation related?
   c. How are the energy and wavelength of electromagnetic radiation related?

7. Which theory of light—the wave or particle theory—best explains the following phenomena?
   a. the interference of light
   b. the photoelectric effect
   c. the emission of electromagnetic radiation by an excited atom

8. Distinguish between the ground state and an excited state of an atom.

9. According to Bohr's model, how is hydrogen's emission spectrum produced?

PRACTICE PROBLEMS

10. Determine the frequency of light whose wavelength is 4.257×10^{-7} cm.

11. Determine the energy in joules of a photon whose frequency is 3.55×10^{17} Hz.

12. Using the two equations $E = h\nu$ and $c = \lambda\nu$, derive an equation expressing E in terms of h, c, and λ.

13. How long would it take a radio wave whose frequency is 7.25×10^{5} Hz to travel from Mars to Earth if the distance between the two planets is approximately 8.00×10^{7} km?

14. Cobalt-60 is an artificial radioisotope that is produced in a nuclear reactor and is used as a gamma-ray source in the treatment of certain types of cancer. If the wavelength of the gamma radiation from a cobalt-60 source is 1.00×10^{-3} nm, calculate the energy of a photon of this radiation.

SECTION 2

The Quantum Model of the Atom

REVIEWING MAIN IDEAS

15. Describe two major shortcomings of Bohr's model of the atom.

16. **a.** What is the principal quantum number?
    b. How is it symbolized?
    c. What are shells?
    d. How does n relate to the number of electrons allowed per main energy level?

17. **a.** What information is given by the angular momentum quantum number?
    b. What are sublevels, or subshells?

18. For each of the following values of n, indicate the numbers and types of sublevels possible for that main energy level. (Hint: See **Figure 2.6.**)
    a. $n = 1$
    b. $n = 2$
    c. $n = 3$
    d. $n = 4$
    e. $n = 7$ (number only)

19. **a.** What information is given by the magnetic quantum number?
    b. How many orbital orientations are possible in each of the s, p, d, and f sublevels?
    c. Explain and illustrate the notation for distinguishing between the different p orbitals in a sublevel.

20. **a.** What is the relationship between n and the total number of orbitals in a main energy level?
b. How many total orbitals are contained in the third main energy level? in the fifth?

21. **a.** What information is given by the spin quantum number?
b. What are the possible values for this quantum number?

22. How many electrons could be contained in the following main energy levels with n equal to the number provided?
a. 1
b. 3
c. 4
d. 6
e. 7

PRACTICE PROBLEMS

23. Sketch the shape of an s orbital and a p orbital.

24. How does a $2s$ orbital differ from a $1s$ orbital?

25. How do a $2p_x$ and a $2p_y$ orbital differ?

SECTION 3

Electron Configurations

REVIEWING MAIN IDEAS

26. **a.** In your own words, state the Aufbau principle.
b. Explain the meaning of this principle in terms of an atom with many electrons.

27. **a.** In your own words, state Hund's rule.
b. What is the basis for this rule?

28. **a.** In your own words, state the Pauli exclusion principle.
b. What is the significance of the spin quantum number?

29. **a.** What is meant by the highest occupied energy level in an atom?
b. What are inner-shell electrons?

30. Determine the highest occupied energy level in the following elements:
a. He
b. Be
c. Al
d. Ca
e. Sn

31. Write the orbital notation for the following elements. (Hint: See Sample Problem A.)
a. P
b. B
c. Na
d. O

32. Write the electron-configuration notation for the element whose atoms contain the following number of electrons:
a. 3
b. 6
c. 8
d. 13

33. Given that the electron configuration for oxygen is $1s^2 2s^2 2p^4$, answer the following questions:
a. How many electrons are in each oxygen atom?
b. What is the atomic number of this element?
c. Write the orbital notation for oxygen's electron configuration.
d. How many unpaired electrons does oxygen have?
e. What is the highest occupied energy level?
f. How many inner-shell electrons does the atom contain?
g. In which orbital(s) are these inner-shell electrons located?

34. **a.** What are the noble gases?
b. What is a noble-gas configuration?
c. How does noble-gas notation simplify writing an atom's electron configuration?

35. Write the noble-gas notation for the electron configuration of each of the elements below. (Hint: See Sample Problem B.)
a. Cl
b. Ca
c. Se

36. **a.** What information is given by the noble-gas notation [Ne]$3s^2$?
b. What element does this represent?

37. Write both the complete electron-configuration notation and the noble-gas notation for each of the elements below. (Hint: See Sample Problem C.)
a. Na
b. Sr
c. P

38. Identify each of the following atoms on the basis of its electron configuration:
a. $1s^2 2s^2 2p^1$
b. $1s^2 2s^2 2p^5$
c. $[Ne]3s^2$
d. $[Ne]3s^2 3p^2$
e. $[Ne]3s^2 3p^5$
f. $[Ar]4s^1$
g. $[Ar]3d^6 4s^2$

PRACTICE PROBLEMS

39. List the order in which orbitals generally fill, from the $1s$ to the $7p$ orbital.

40. Write the noble-gas notation for the electron configurations of each of the following elements:
a. As
b. Pb
c. Lr
d. Hg
e. Sn
f. Xe
g. La

41. How do the electron configurations of chromium and copper contradict the Aufbau principle?

Mixed Review

REVIEWING MAIN IDEAS

42. **a.** Which has a longer wavelength: green light or yellow light?
b. Which has a higher frequency: an x-ray or a microwave?
c. Which travels at a greater speed: ultraviolet light or infrared light?

43. Write both the complete electron-configuration and noble-gas notation for each of the following:
a. Ar **b.** Br **c.** Al

44. Given the speed of light as 3.00×10^8 m/s, calculate the wavelength of the electromagnetic radiation whose frequency is 7.500×10^{12} Hz.

45. **a.** What is the electromagnetic spectrum?
b. What units can be used to express wavelength?
c. What unit is used to express frequencies of electromagnetic waves?

46. Given that the electron configuration for phosphorus is $1s^2 2s^2 2p^6 3s^2 3p^3$, answer the following questions:
a. How many electrons are in each atom?
b. What is the atomic number of this element?
c. Write the orbital notation for this element.
d. How many unpaired electrons does an atom of phosphorus have?
e. What is its highest occupied energy level?
f. How many inner-shell electrons does the atom contain?
g. In which orbital(s) are these inner-shell electrons located?

47. What is the frequency of a radio wave whose energy is 1.55×10^{-24} J per photon?

48. Write the noble-gas notation for the electron configurations of each of the following elements:
a. Hf
b. Sc
c. Fe
d. At
e. Ac
f. Zn

49. Describe the major similarities and differences between Schrödinger's model of the atom and the model proposed by Bohr.

50. When sodium is heated, a yellow spectral line whose energy is 3.37×10^{-19} J per photon is produced.
a. What is the frequency of this light?
b. What is the wavelength of this light?

51. **a.** What is an orbital?
b. Describe an orbital in terms of an electron cloud.

CRITICAL THINKING

52. **Inferring Relationships** In the emission spectrum of hydrogen shown in **Figure 1.5**, each colored line is produced by the emission of photons with specific energies. Substances also produce absorption spectra when electromagnetic radiation passes through them. Certain wavelengths are absorbed. Using the diagram below, predict what the wavelengths of the absorption lines will be when white light (all of the colors of the visible spectrum) is passed through hydrogen gas.

Hydrogen absorption spectrum

53. Applying Models In discussions of the photoelectric effect, the minimum energy needed to remove an electron from the metal is called the *threshold energy* and is a characteristic of the metal. For example, chromium, Cr, will emit electrons when the wavelength of the radiation is 284 nm or less. Calculate the threshold energy for chromium. (Hint: You will need to use the two equations that describe the relationships between wavelength, frequency, speed of light, and Planck's constant.)

54. Analyzing Information Four electrons in an atom have the four sets of quantum numbers given below. Which electrons are in the same orbital? Explain your answer.

a. 1, 0, 0, −1/2
b. 1, 0, 0, +1/2
c. 2, 1, 1, +1/2
d. 2, 1, 0, +1/2

55. Relating Ideas Which of the sets of quantum numbers below are possible? Which are impossible? Explain your choices.

a. 2, 2, 1, +1/2
b. 2, 0, 0, −1/2
c. 2, 0, 1, −1/2

USING THE HANDBOOK

56. Sections 1 and 2 of the *Elements Handbook* (Appendix A) contain information on an analytical test and a technological application for Group 1 and 2 elements. The test and application are based on the emission of light from atoms. Review these sections to answer the following:

a. What analytical technique utilizes the emission of light from excited atoms?
b. What elements in Groups 1 and 2 can be identified by this technique?
c. What types of compounds are used to provide color in fireworks?
d. What wavelengths within the visible spectrum would most likely contain emission lines for barium?

RESEARCH AND WRITING

57. Neon signs do not always contain neon gas. The various colored lights produced by the signs are due to the emission of a variety of low-pressure gases in different tubes. Research other kinds of gases used in neon signs, and list the colors that they emit.

58. Prepare a report about the photoelectric effect, and cite some of its practical uses. Explain the basic operation of each device or technique mentioned.

ALTERNATIVE ASSESSMENT

59. Performance A spectroscope is a device used to produce and analyze spectra. Construct a simple spectroscope, and determine the absorption spectra of several elemental gases. (Your teacher will provide you with the gas discharge tubes containing samples of different gases.)