CHAPTER 3 Summary

BIG IDEA Vectors can be used to represent and predict the two-dimensional motion of an object.

SECTION 1 Introduction to Vectors

- A scalar is a quantity completely specified by only a number with appropriate units, whereas a vector is a quantity that has magnitude and direction.
- Vectors can be added graphically using the triangle method of addition, in which the tail of one vector is placed at the head of the other. The resultant is the vector drawn from the tail of the first vector to the head of the last vector.

KEY TERMS

scalar
vector
resultant

SECTION 2 Vector Operations

- The Pythagorean theorem and the inverse tangent function can be used to find the magnitude and direction of a resultant vector.
- Any vector can be resolved into its component vectors by using the sine and cosine functions.

KEY TERM

components of a vector

SECTION 3 Projectile Motion

- Neglecting air resistance, a projectile has a constant horizontal velocity and a constant downward free-fall acceleration.
- In the absence of air resistance, projectiles follow a parabolic path.

KEY TERM

projectile motion

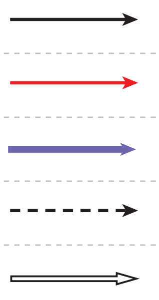
SECTION 4 Relative Motion

- If the frame of reference is denoted with subscripts ($\mathbf{v}_{ab}$ is the velocity of object or frame *a* with respect to object or frame *b*), then the velocity of an object with respect to a different frame of reference can be found by adding the known velocities so that the subscript starts with the letter that ends the preceding velocity subscript: $\mathbf{v}_{ac} = \mathbf{v}_{ab} + \mathbf{v}_{bc}$.
- If the order of the subscripts is reversed, there is a change in sign; for example, $\mathbf{v}_{cd} = -\mathbf{v}_{dc}$.

VARIABLE SYMBOLS

Quantities		Units	
d (vector)	displacement	m	meters
v (vector)	velocity	m/s	meters /second
a (vector)	acceleration	m/s^2	meters /second^2
Δx (scalar)	horizontal component	m	meters
Δy (scalar)	vertical component	m	meters

DIAGRAM SYMBOLS



displacement vector
velocity vector
acceleration vector
resultant vector
component

Problem Solving

See **Appendix D: Equations** for a summary of the equations introduced in this chapter. If you need more problem-solving practice, see **Appendix I: Additional Problems.**

CHAPTER 3 Review

Vectors and the Graphical Method

REVIEWING MAIN IDEAS

1. The magnitude of a vector is a scalar. Explain this statement.

2. If two vectors have unequal magnitudes, can their sum be zero? Explain.

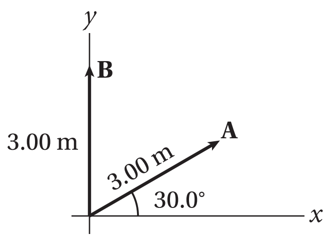
3. What is the relationship between instantaneous speed and instantaneous velocity?

4. What is another way of saying −30 m/s west?

5. Is it possible to add a vector quantity to a scalar quantity? Explain.

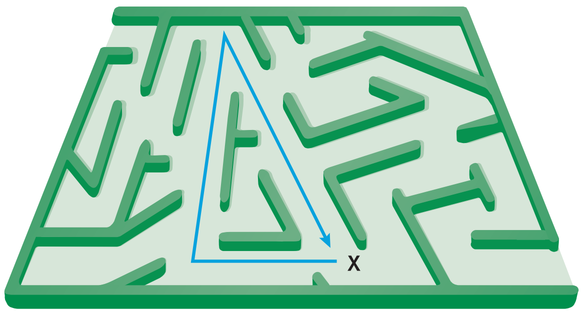
6. Vector **A** is 3.00 units in length and points along the positive x-axis. Vector **B** is 4.00 units in length and points along the negative y-axis. Use graphical methods to find the magnitude and direction of the following vectors:
   a. **A** + **B**
   b. **A** − **B**
   c. **A** + 2**B**
   d. **B** − **A**

7. Each of the displacement vectors **A** and **B** shown in the figure below has a magnitude of 3.00 m. Graphically find the following:
   a. **A** + **B**
   b. **A** − **B**
   c. **B** − **A**
   d. **A** − 2**B**



8. A dog searching for a bone walks 3.50 m south, then 8.20 m at an angle of 30.0° north of east, and finally 15.0 m west. Use graphical techniques to find the dog's resultant displacement vector.

9. A man lost in a maze makes three consecutive displacements so that at the end of the walk he is back where he started, as shown below. The first displacement is 8.00 m westward, and the second is 13.0 m northward. Use the graphical method to find the third displacement.



CONCEPTUAL QUESTIONS

10. If **B** is added to **A**, under what conditions does the resultant have the magnitude equal to $A + B$?

11. Give an example of a moving object that has a velocity vector and an acceleration vector in the same direction and an example of one that has velocity and acceleration vectors in opposite directions.

12. A student accurately uses the method for combining vectors. The two vectors she combines have magnitudes of 55 and 25 units. The answer that she gets is either 85, 20, or 55. Pick the correct answer, and explain why it is the only one of the three that can be correct.

13. If a set of vectors laid head to tail forms a closed polygon, the resultant is zero. Is this statement true? Explain your reasoning.

Vector Operations

REVIEWING MAIN IDEAS

14. Can a vector have a component equal to zero and still have a nonzero magnitude?

15. Can a vector have a component greater than its magnitude?

16. Explain the difference between vector addition and vector resolution.

17. How would you add two vectors that are not perpendicular or parallel?

CONCEPTUAL QUESTIONS

18. If **A** + **B** equals 0, what can you say about the components of the two vectors?

19. Under what circumstances would a vector have components that are equal in magnitude?

20. The vector sum of three vectors gives a resultant equal to zero. What can you say about the vectors?

PRACTICE PROBLEMS

For problems 21–23, see Sample Problem A.

21. A girl delivering newspapers travels three blocks west, four blocks north, and then six blocks east.
 a. What is her resultant displacement?
 b. What is the total distance she travels?

22. A quarterback takes the ball from the line of scrimmage, runs backward for 10.0 yards, and then runs sideways parallel to the line of scrimmage for 15.0 yards. At this point, he throws a 50.0-yard forward pass straight down the field. What is the magnitude of the football's resultant displacement?

23. A shopper pushes a cart 40.0 m south down one aisle and then turns 90.0° and moves 15.0 m. He then makes another 90.0° turn and moves 20.0 m. Find the shopper's total displacement. (There could be more than one correct answer.)

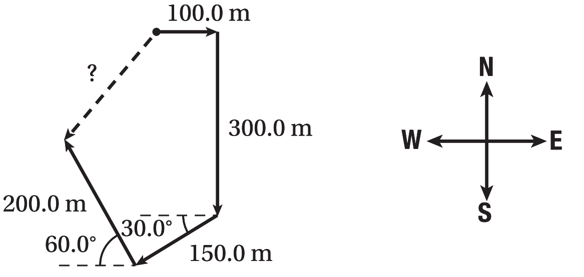
For problems 24–25, see Sample Problem B.

24. A submarine dives 110.0 m at an angle of 10.0° below the horizontal. What are the two components?

25. A person walks 25.0° north of east for 3.10 km. How far would another person walk due north and due east to arrive at the same location?

For problem 26, see Sample Problem C.

26. A person walks the path shown below. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?



Projectile Motion

REVIEWING MAIN IDEAS

27. A dart is fired horizontally from a dart gun, and another dart is dropped simultaneously from the same height. If air resistance can be neglected, which dart hits the ground first?

28. If a rock is dropped from the top of a sailboat's mast, will it hit the deck at the same point whether the boat is at rest or in motion at constant velocity?

29. Does a ball dropped out of the window of a moving car take longer to reach the ground than one dropped at the same height from a car at rest?

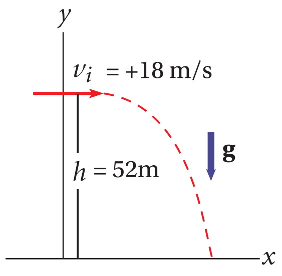
30. A rock is dropped at the same instant that a ball at the same elevation is thrown horizontally. Which will have the greater speed when it reaches ground level?

PRACTICE PROBLEMS

For problems 31–33, see Sample Problem D.

31. In 1974, Nolan Ryan broke the existing record for the fastest pitch thrown in Major League Baseball. If this pitch were thrown horizontally, the ball would fall 0.809 m (2.65 ft) by the time it reached home plate, 18.3 m (60 ft) away. How fast was Ryan's pitch?

32. A person standing at the edge of a seaside cliff kicks a stone over the edge with a speed of 18 m/s. The cliff is 52 m above the water's surface, as shown at right. How long does it take for the stone to fall to the water? With what speed does it strike the water?



33. A spy in a speed boat is being chased down a river by government officials in a faster craft. Just as the officials' boat pulls up next to the spy's boat, both boats reach the edge of a 5.0 m waterfall. If the spy's speed is 15 m/s and the officials' speed is 26 m/s, how far apart will the two vessels be when they land below the waterfall?

For problems 34 –37, see Sample Problem E.

34. A shell is fired from the ground with an initial speed of 1.70×10^3 m/s (approximately five times the speed of sound) at an initial angle of 55.0° to the horizontal. Neglecting air resistance, find

a. the shell's horizontal range

b. the amount of time the shell is in motion

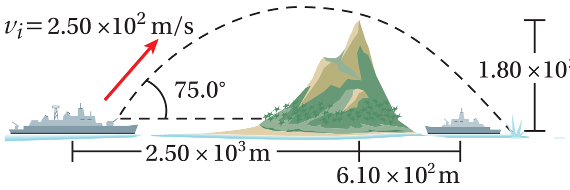
35. A place kicker must kick a football from a point 36.0 m (about 40.0 yd) from the goal. As a result of the kick, the ball must clear the crossbar, which is 3.05 m high. When kicked, the ball leaves the ground with a speed of 20.0 m/s at an angle of 53° to the horizontal.

a. By how much does the ball clear or fall short of clearing the crossbar?

b. Does the ball approach the crossbar while still rising or while falling?

36. When a water gun is fired while being held horizontally at a height of 1.00 m above ground level, the water travels a horizontal distance of 5.00 m. A child, who is holding the same gun in a horizontal position, is also sliding down a 45.0° incline at a constant speed of 2.00 m/s. If the child fires the gun when it is 1.00 m above the ground and the water takes 0.329 s to reach the ground, how far will the water travel horizontally?

37. A ship maneuvers to within 2.50×10^3 m of an island's 1.80×10^3 m high mountain peak and fires a projectile at an enemy ship 6.10×10^2 m on the other side of the peak, as illustrated below. If the ship shoots the projectile with an initial velocity of 2.50×10^2 m/s at an angle of 75.0°, how close to the enemy ship does the projectile land? How close (vertically) does the projectile come to the peak?



Relative Motion

REVIEWING MAIN IDEAS

38. Explain the statement "All motion is relative."

39. What is a frame of reference?

40. When we describe motion, what is a common frame of reference?

41. A small airplane is flying at 50 m/s toward the east. A wind of 20 m/s toward the east suddenly begins to blow and gives the plane a velocity of 70 m/s east.

a. Which vector is the resultant vector?

b. What is the magnitude of the wind velocity?

42. A ball is thrown upward in the air by a passenger on a train that is moving with constant velocity.

a. Describe the path of the ball as seen by the passenger. Describe the path as seen by a stationary observer outside the train.

b. How would these observations change if the train were accelerating along the track?

PRACTICE PROBLEMS

For problems 43–46, see Sample Problem F.

43. A river flows due east at 1.50 m/s. A boat crosses the river from the south shore to the north shore by maintaining a constant velocity of 10.0 m/s due north relative to the water.

a. What is the velocity of the boat as viewed by an observer on shore?

b. If the river is 325 m wide, how far downstream is the boat when it reaches the north shore?

44. The pilot of an aircraft wishes to fly due west in a 50.0 km/h wind blowing toward the south. The speed of the aircraft in the absence of a wind is 205 km/h.
a. In what direction should the aircraft head?
b. What should its speed relative to the ground be?

45. A hunter wishes to cross a river that is 1.5 km wide and that flows with a speed of 5.0 km/h. The hunter uses a small powerboat that moves at a maximum speed of 12 km/h with respect to the water. What is the minimum time necessary for crossing?

46. A swimmer can swim in still water at a speed of 9.50 m/s. He intends to swim directly across a river that has a downstream current of 3.75 m/s.
a. What must the swimmer's direction be?
b. What is his velocity relative to the bank?

Mixed Review

47. A ballplayer hits a home run, and the baseball just clears a wall 21.0 m high located 130.0 m from home plate. The ball is hit at an angle of 35.0° to the horizontal, and air resistance is negligible. Assume the ball is hit at a height of 1.0 m above the ground.
a. What is the initial speed of the ball?
b. How much time does it take for the ball to reach the wall?
c. Find the components of the velocity and the speed of the ball when it reaches the wall.

48. A daredevil jumps a canyon 12 m wide. To do so, he drives a car up a 15° incline.
a. What minimum speed must he achieve to clear the canyon?
b. If the daredevil jumps at this minimum speed, what will his speed be when he reaches the other side?

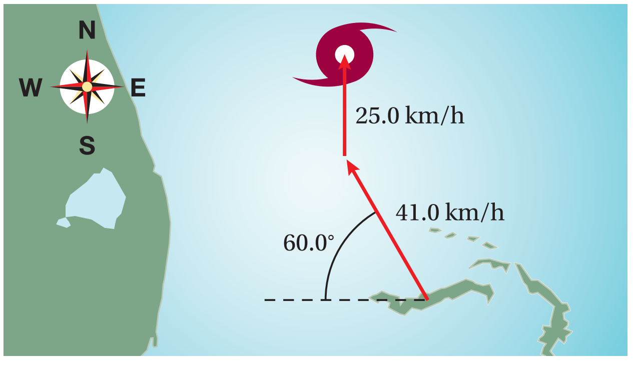
49. A 2.00 m tall basketball player attempts a goal 10.00 m from the basket (3.05 m high). If he shoots the ball at a 45.0° angle, at what initial speed must he throw the basketball so that it goes through the hoop without striking the backboard?

50. An escalator is 20.0 m long. If a person stands on the escalator, it takes 50.0 s to ride to the top.
a. If a person walks up the moving escalator with a speed of 0.500 m/s relative to the escalator, how long does it take the person to get to the top?
b. If a person walks down the "up" escalator with the same relative speed as in item (a), how long does it take to reach the bottom?

51. A ball is projected horizontally from the edge of a table that is 1.00 m high, and it strikes the floor at a point 1.20 m from the base of the table.
a. What is the initial speed of the ball?
b. How high is the ball above the floor when its velocity vector makes a 45.0° angle with the horizontal?

52. How long does it take an automobile traveling 60.0 km/h to become even with a car that is traveling in another lane at 40.0 km/h if the cars' front bumpers are initially 125 m apart?

53. The eye of a hurricane passes over Grand Bahama Island. It is moving in a direction 60.0° north of west with a speed of 41.0 km/h. Exactly three hours later, the course of the hurricane shifts due north, and its speed slows to 25.0 km/h, as shown below. How far from Grand Bahama is the hurricane 4.50 h after it passes over the island?



54. A boat moves through a river at 7.5 m/s relative to the water, regardless of the boat's direction. If the water in the river is flowing at 1.5 m/s, how long does it take the boat to make a roundtrip consisting of a 250 m displacement downstream followed by a 250 m displacement upstream?

55. A car is parked on a cliff overlooking the ocean on an incline that makes an angle of 24.0° below the horizontal. The negligent driver leaves the car in neutral, and the emergency brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s^2 and travels 50.0 m to the edge of the cliff. The cliff is 30.0 m above the ocean.
- **a.** What is the car's position relative to the base of the cliff when the car lands in the ocean?
- **b.** How long is the car in the air?

56. A golf ball with an initial angle of 34° lands exactly 240 m down the range on a level course.
- **a.** Neglecting air friction, what initial speed would achieve this result?
- **b.** Using the speed determined in item (a), find the maximum height reached by the ball.

57. A car travels due east with a speed of 50.0 km/h. Rain is falling vertically with respect to Earth. The traces of the rain on the side windows of the car make an angle of 60.0° with the vertical. Find the velocity of the rain with respect to the following:
- **a.** the car
- **b.** Earth

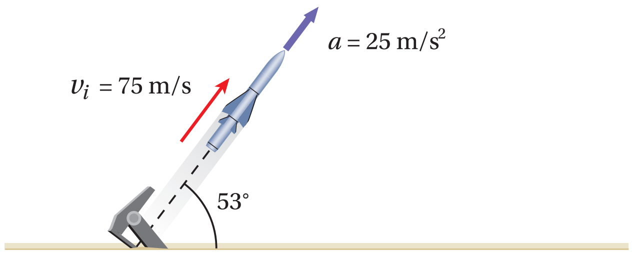
58. A shopper in a department store can walk up a stationary escalator in 30.0 s. If the escalator can carry the standing shopper to the next floor in 20.0 s when it is moving, how long would it take the shopper to walk up the moving escalator? Assume the same walking effort for the shopper whether the escalator is stopped or moving.

59. If a person can jump a horizontal distance of 3.0 m on Earth, how far could the person jump on the moon, where the free-fall acceleration is $g/6$ and $g = 9.81\ m/s^2$? How far could the person jump on Mars, where the acceleration due to gravity is $0.38g$?

60. A science student riding on a flatcar of a train moving at a constant speed of 10.0 m/s throws a ball toward the caboose along a path that the student judges as making an initial angle of 60.0° with the horizontal. The teacher, who is standing on the ground nearby, observes the ball rising vertically. How high does the ball rise?

61. A football is thrown directly toward a receiver with an initial speed of 18.0 m/s at an angle of 35.0° above the horizontal. At that instant, the receiver is 18.0 m from the quarterback. In what direction and with what constant speed should the receiver run to catch the football at the level at which it was thrown?

62. A rocket is launched at an angle of 53° above the horizontal with an initial speed of 75 m/s, as shown below. It moves for 25 s along its initial line of motion with an acceleration of 25 m/s^2. At this time, its engines fail, and the rocket proceeds to move as a free body.
- **a.** What is the rocket's maximum altitude?
- **b.** What is the rocket's total time of flight?
- **c.** What is the rocket's horizontal range?



ALTERNATIVE ASSESSMENT

1. Work in cooperative groups to analyze a game of chess in terms of displacement vectors. Make a model chessboard, and draw arrows showing all the possible moves for each piece as vectors made of horizontal and vertical components. Then have two members of your group play the game while the others keep track of each piece's moves. Be prepared to demonstrate how vector addition can be used to explain where a piece would be after several moves.

2. Use a garden hose to investigate the laws of projectile motion. Design experiments to investigate how the angle of the hose affects the range of the water stream. (Assume that the initial speed of water is constant and is determined by the pressure indicated by the faucet's setting.) What quantities will you measure, and how will you measure them? What variables do you need to control? What is the shape of the water stream? How can you reach the maximum range? How can you reach the highest point? Present your results to the rest of the class and discuss the conclusions.

3. You are helping NASA engineers design a basketball court for a colony on the moon. How do you anticipate the ball's motion compared with its motion on Earth? What changes will there be for the players—how they move and how they throw the ball? What changes would you recommend for the size of the court, the basket height, and other regulations in order to adapt the sport to the moon's low gravity? Create a presentation or a report presenting your suggestions, and include the physics concepts behind your recommendations.

4. There is conflicting testimony in a court case. A police officer claims that his radar monitor indicated that a car was traveling at 176 km/h (110 mi/h). The driver argues that the radar must have recorded the relative velocity because he was only going 88 km/h (55 mi/h). Is it possible that both are telling the truth? Could one be lying? Prepare scripts for expert witnesses, for both the prosecution and the defense, that use physics to justify their positions before the jury. Create visual aids to be used as evidence to support the different arguments.

GRAPHING CALCULATOR PRACTICE

Two-Dimensional Motion

Recall the following equation from your studies of projectiles launched at an angle.

$$\Delta y = (v_i \sin \theta)\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

Consider a baseball that is thrown straight up in the air. The equation for projectile motion can be entered as Y_1 on a graphing calculator.

$$Y_1 = VX - 4.9X^2$$

Given the initial velocity (V), your graphing calculator can calculate the height (Y_1) of the baseball versus the time interval (X) that the ball remains in the air. Why is the factor sin θ missing from the equation for Y_1?

In this activity, you will determine the maximum height and flight time of a baseball thrown vertically at various initial velocities.

Go online to HMHScience.com to find the skillsheet and program for this graphing calculator activity.