Math Tutor

Calculating with Half-Life

The rate at which a sample of a radioactive nuclide decays is expressed in terms of half-life. This quantity is the time required for half of the atoms of a sample of a given nuclide to decay. For example, it takes 37.2 min for half of the nuclei of chlorine-38 to decay to argon-38. After 37.2 min, 0.50 g of a 1.0 g sample of chlorine-38 will remain, and there will be 0.50 g of argon-38. After two half-lives (74.4 min), the fraction of chlorine-38 that remains will be $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$.

After n half-lives, the fraction of a radioactive nuclide that remains is $\left(\frac{1}{2}\right)^n$, or 2^{-n}.

If you know the amount of nuclide that was present initially and the amount of nuclide that remains, you can determine the number of half-lives that have passed.

Problem-Solving TIPS

- **Familiarize yourself with the values of some common powers of two (2^n, $n = 1, 2, 3, 4, 5, 6$, etc). This will allow you to determine the number of half-lives quickly.**

Sample Problem

The half-life of polonium-218 is 3.04 min. A sample of polonium contains 0.00558 g of $^{218}_{84}Po$. What mass of $^{218}_{84}Po$ will remain after 18.24 min?

First, you must determine the number of half-lives that have passed in 18.24 min.

$$\text{number of half-lives} = \frac{\text{time elapsed}}{\text{half-life}} = \frac{18.24 \text{ min}}{3.04 \text{ min}} = 6.00 \text{ half-lives}$$

Then, to determine the mass of polonium-218 remaining, apply the following relationship:

$$\text{mass remaining} = \text{starting mass} \times \text{fraction remaining}$$

$$\text{mass } ^{218}_{84}\text{Po remaining} = 0.00558 \text{ g} \times \left(\frac{1}{2}\right)^6 = 0.00558 \text{ g} \times \frac{1}{64} = 8.72 \times 10^{-5} \text{ g } ^{218}_{84}\text{Po}$$

The half-life of potassium-40 is 1.3×10^9 years. A volcanic rock contains $\frac{1}{8}$ of the amount of potassium-40 found in newly formed rocks. When was the rock formed?

First, determine the number of half-lives that have passed.

$$\text{fraction remaining} = \frac{1}{8} = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$$

Therefore, three half-lives have passed. The time since the rock was formed is

$$3 \text{ half-lives} \times \left(1.3 \times 10^9 \frac{\text{y}}{\text{half-life}}\right) = 3.9 \times 10^9 \text{ y.}$$

Practice

1. A sample of chromium contains 8.9×10^{-5} g of the radioactive nuclide chromium-51, which has a half-life of 28 days. What mass of chromium-51 will remain in the sample after 168 days?
2. The half-life of lead-202 is 53 000 years. A sample of lead contains only $\frac{1}{256}$ of the expected amount of lead-202. How old is the lead sample?

CHAPTER 21 Summary

BIG IDEA When an atomic nucleus becomes unstable, it emits radiation in the form of particles and energy to regain stability. Nuclei often change identity during radioactive decay.

SECTION 1 The Nucleus

- The difference between the sum of the masses of the nucleons and electrons in an atom and the actual mass of an atom is the mass defect, or nuclear binding energy.
- Nuclear stability tends to be greatest when nucleons are paired, when there are magic numbers of nucleons, and when there are certain neutron-proton ratios.
- Nuclear reactions, which are represented by nuclear equations, can involve the transmutation of nuclides.

KEY TERMS

nucleon
nuclide
mass defect
nuclear binding energy
nuclear shell model
magic number
nuclear reaction
transmutation

SECTION 2 Radioactive Decay

- Radioactive nuclides decay to form more stable nuclides.
- Alpha, beta, positron, and gamma emissions are types of radioactive decay. Electron capture is also a type of radioactive decay. The type of decay is related to the nucleon content and the energy level of the nucleus.
- The half-life of a radioactive nuclide is the length of time that it takes for half of a given number of atoms of the nuclide to decay.
- Artificial transmutations are used to produce artificial radioactive nuclides, which include the transuranium elements.

KEY TERMS

radioactive decay
nuclear radiation
radioactive nuclide
alpha particle
beta particle
positron
electron capture
gamma ray
half-life
decay series
parent nuclide
daughter nuclide
artificial transmutation
transuranium element

SECTION 3 Nuclear Radiation

- Alpha particles, beta particles, and gamma rays have different penetrating abilities and shielding requirements.
- Film badges, Geiger-Müller counters, and scintillation detectors are used to detect radiation.
- Everyone is exposed to environmental background radiation.
- Radioactive nuclides have many uses, including radioactive dating, disease detection, and therapy.
- Nuclear waste must be contained, stored, and disposed of in a way that minimizes harm to people and the environment.

KEY TERMS

roentgen
rem
film badge
Geiger-Müller counter
scintillation counter
radioactive dating
radioactive tracer
nuclear waste

SECTION 4 Nuclear Fission and Nuclear Fusion

- Nuclear fission and nuclear fusion are nuclear reactions in which the splitting and fusing of nuclei produce more stable nuclei and release enormous amounts of energy.
- Controlled fission reactions produce energy and radioactive nuclides.
- Fusion reactions produce the sun's energy in the form of heat and light. If fusion reactions could be controlled, they would produce more usable energy per gram of fuel than fission reactions.

KEY TERMS

nuclear fission
chain reaction
critical mass
nuclear reactor
nuclear power plant
shielding
control rod
moderator
nuclear fusion

CHAPTER 21 Review

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SECTION 1

The Nucleus

REVIEWING MAIN IDEAS

1. **a.** How does mass defect relate to nuclear binding energy?
   b. How does binding energy per nucleon vary with mass number?
   c. How does binding energy per nucleon affect the stability of a nucleus?

2. Describe three ways in which the number of protons and the number of neutrons in a nucleus affect the stability of the nucleus.

PRACTICE PROBLEMS

3. The mass of a $^{20}_{10}$Ne atom is 19.992 44 u. Calculate the atom's mass defect.

4. The mass of a $^{7}_{3}$Li atom is 7.016 00 u. Calculate the atom's mass defect.

5. Calculate the nuclear binding energy of one lithium-6 atom. The measured atomic mass of lithium-6 is 6.015 u.

6. Calculate the binding energies of the following two nuclei, and indicate which nucleus releases more energy when formed. You will need information from the periodic table and the text.
   a. atomic mass 34.988011 u, $^{35}_{19}$K
   b. atomic mass 22.989767 u, $^{23}_{11}$Na

7. **a.** What is the binding energy per nucleon for each nucleus in the previous problem?
   b. Which nucleus is more stable?

8. The mass of $^{7}_{3}$Li is 7.016 00 u. Calculate the binding energy per nucleon for $^{7}_{3}$Li.

9. Calculate the neutron-proton ratios for the following nuclides:
   a. $^{12}_{6}$C
   b. $^{3}_{1}$H
   c. $^{206}_{82}$Pb
   d. $^{134}_{50}$Sn

10. **a.** Locate the nuclides in problem 9 on the graph in **Figure 1.2.** Which ones lie within the band of stability?
    b. For the stable nuclides, determine whether their neutron-proton ratio tends toward 1:1 or 1.5:1.

11. Balance the following nuclear equations. (Hint: See Sample Problem A.)
    a. $^{43}_{19}\text{K} \longrightarrow {}^{43}_{20}\text{Ca} + \underline{\ ?\ }$
    b. $^{233}_{92}\text{U} \longrightarrow {}^{229}_{90}\text{Th} + \underline{\ ?\ }$
    c. $^{11}_{6}\text{C} + \underline{\ ?\ } \longrightarrow {}^{11}_{5}\text{B}$
    d. $^{13}_{7}\text{N} \longrightarrow + {}^{1}_{0}\beta^{+} + \underline{\ ?\ }$

12. Write the nuclear equation for the release of an alpha particle by $^{210}_{84}$Po.

13. Write the nuclear equation for the release of a beta particle by $^{210}_{82}$Pb.

SECTION 2

Radioactive Decay

REVIEWING MAIN IDEAS

14. Beyond what point on the periodic table are all of the naturally occuring elements radioactive?

15. What changes in atomic number and mass number occur in each of the following types of radioactive decay?
    a. alpha emission
    b. beta emission
    c. positron emission
    d. electron capture

16. Which types of radioactive decay cause the transmutation of a nuclide? (Hint: Review the definition of *transmutation*.)

17. Explain how beta emission, positron emission, and electron capture affect the neutron-proton ratio.

18. Write the nuclear reactions that show particle conversion for the following types of radioactive decay:
    a. beta emission
    b. positron emission
    c. electron capture

19. Compare electrons, beta particles, and positrons.

20. **a.** What are gamma rays?
    b. How do scientists think gamma rays are produced?

21. How does the half-life of a nuclide relate to the stability of the nuclide?

22. List the three parent nuclides of the natural decay series.

23. How are artificial radioactive isotopes produced?

24. Neutrons are more effective for bombarding atomic nuclei than protons or alpha particles are. Why?

25. Why are all of the transuranium elements radioactive? (Hint: See Section 1.)

PRACTICE PROBLEMS

26. The half-life of plutonium-239 is 24 110 years. Of an original mass of 100. g, how much plutonium-239 remains after 96 440 years? (Hint: See Sample Problem B.)

27. The half-life of thorium-227 is 18.72 days. How many days are required for three-fourths of a given amount of thorium-227 to decay?

28. Exactly $\frac{1}{16}$ of a given amount of protactinium-234 remains after 26.76 hours. What is the half-life of protactinium-234?

29. How many milligrams of a 15.0 mg sample of radium-226 remain after 6396 years? The half-life of radium-226 is 1599 years.

SECTION 3

Nuclear Radiation

REVIEWING MAIN IDEAS

30. Why can a radioactive material affect photographic film even though the film is completely wrapped in black paper?

31. How does the penetrating ability of gamma rays compare with that of alpha particles and beta particles?

32. How does nuclear radiation damage biological tissue?

33. Explain how film badges, Geiger-Müller counters, and scintillation detectors are used to detect radiation and measure radiation exposure.

34. How is the age of an object that contains a radioactive nuclide estimated?

SECTION 4

Nuclear Fission and Nuclear Fusion

REVIEWING MAIN IDEAS

35. How is the fission of a uranium-235 nucleus induced?

36. How does the fission of uranium-235 produce a chain reaction?

37. Describe the purposes of the five major components of a nuclear power plant.

38. Describe the reaction that produces the sun's energy.

39. What is one problem that must be overcome before controlled fusion reactions that produce energy are a reality?

Mixed Review

REVIEWING MAIN IDEAS

40. Balance the following nuclear reactions:
    - **a.** ${}^{239}_{93}\mathrm{Np} \longrightarrow {}^{0}_{-1}\beta + \underline{\ ?\ }$
    - **b.** ${}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \longrightarrow \underline{\ ?\ }$
    - **c.** ${}^{32}_{15}\mathrm{P} + \underline{\ ?\ } \longrightarrow {}^{33}_{15}\mathrm{P}$
    - **d.** ${}^{236}_{92}\mathrm{U} \longrightarrow {}^{94}_{36}\mathrm{Kr} + \underline{\ ?\ } + 3\,{}^{1}_{0}n$

41. After 4797 years, how much of the original 0.250 g of radium-226 remains? The half-life of radium-226 is 1599 years.

42. The parent nuclide of the thorium decay series is ${}^{232}_{90}\mathrm{Th}$. The first four decays are as follows: alpha emission, beta emission, beta emission, and alpha emission. Write the nuclear equations for this series of emissions.

43. The half-life of radium-224 is 3.66 days. What was the original mass of radium-224 if 0.0500 g remains after 7.32 days?

44. Calculate the neutron-proton ratios for the following nuclides, and determine where they lie in relation to the band of stability.
    - **a.** ${}^{235}_{92}\mathrm{U}$
    - **b.** ${}^{16}_{8}\mathrm{O}$
    - **c.** ${}^{56}_{26}\mathrm{Fe}$
    - **d.** ${}^{156}_{60}\mathrm{Nd}$

45. Calculate the binding energy per nucleon of ${}^{238}_{92}\mathrm{U}$ in joules. The atomic mass of a ${}^{238}_{92}\mathrm{U}$ nucleus is 238.050 784 u.

46. The energy released by the formation of a nucleus of $^{56}_{26}Fe$ is 7.89×10^{-11} J. Use Einstein's equation $E = mc^2$ to determine how much mass (in kilograms) is lost in this process.

47. Calculate the binding energy for one mole of deuterium atoms. The measured mass of deuterium is 2.0140 u.

CRITICAL THINKING

48. Why do we compare binding energy per nuclear particle of different nuclides instead of the total binding energy per nucleus of different nuclides?

49. Why is the constant rate of decay of radioactive nuclei so important in radioactive dating?

50. Which of the following nuclides of carbon is more likely to be stable? State reasons for your answer.
    a. $^{11}_{6}C$ **b.** $^{12}_{6}C$

51. Which of the following nuclides of iron is more likely to be stable? State reasons for your answer.
    a. $^{56}_{26}Fe$ **b.** $^{59}_{26}Fe$

52. Use the data shown below to determine the following:
    a. the isotopes that would be best for dating ancient rocks
    b. the isotopes that could be used as tracers
    State reasons for your answers.

Element	Half-Life
potassium-40	1.28×10^9 y
potassium-42	12.36 h
uranium-238	4.468×10^9 y
uranium-239	23.47 min

RESEARCH AND WRITING

53. Investigate the history of the Manhattan Project.

54. Research the 2011 Fukashima reactor accident in Japan. What factors combined to cause the accident?

55. Find out about the various fusion-energy research projects that are being conducted in the United States and other parts of the world. What obstacles in finding an economical method of producing energy must still be overcome?

ALTERNATIVE ASSESSMENT

56. Using the library, research the medical uses of radioactive isotopes such as cobalt-60 and technetium-99. Evaluate the benefits and risks of using radioisotopes in the diagnosis and treatment of medical conditions. Report your findings to the class.